A New 2-D Image Reconstruction Algorithm Based on FDTD and Design Sensitivity Analysis

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***Abstract* — In this paper, we propose a numerical algorithm to reconstruct the complex permittivity profile of unknown scatterers by using the FDTD technique and the design sensitivity analysis (DSA). By introducing the DSA and the adjoint variable method, we can calculate the derivatives of error function with respect to complex permittivity variables, and reduce the computational costs. Proposed method is validated by applying to the reconstruction of unknown 2-D scatterers which are illuminated by TM^z with a gaussian pulsed plane wave.**

I. Introduction

The development of methods for the reconstruction of the unknown complex permittivity distribution of scatterers from the measured scattered field has been much attracted over the last years because it is considered to be fundamental and essential in microwave imaging applications. The reconstruction of complex permittivity profile in inhomogeneous structures can be considered as an optimization problem to minimize the difference between the measured field data and the calculated ones by controlling the complex dielectric permittivity in test domain. Such a difference is defined as an error function to be minimized. However, the inverse scattering problems are known to have nonlinear and ill-posed properties due to the lack of the measured information and multi-scattering effects between the objects [1]. In order to effectively reconstruct the unknown profiles, the first order method using the gradient information and the iterative technique has been preferred.

Recently an optimization method based on FDTD and the design sensitivity analysis (DSA) in frequency domain has been proposed [2][3]. The DSA concerns the relationship between the design goal (or the objective function) and design variables. That is, the DSA is to evaluate the derivative of objective function with respect to the design variables.

In this paper, we propose a new reconstruction algorithm that uses the derivatives information calculated by the FDTD technique and DSA. In order to effectively calculate the derivative information of error function, we adopted the adjoint variable method [4]. The proposed method based on the adjoint variable method needs twice the CPU time to solve the forward problem using FDTD as many as the number of complex permittivity variables per iteration. And by introducing the topology optimization based on normalized material density, we can improve the characteristics of convergence.

In order to realize a plane wave source, we introduce the TF/SF technique [5]. To reduce the computational domain, Berenger's PML technique is also adopted.

II. Formulation

A. Problem Definition

To reconstruct the unknown complex permittivity distribution of a scatterer in the objective domain, it is required to minimize the difference between the calculated scattered fields and the measured ones. To evaluate such a difference, we define an error or objective function as

$$F = \frac{1}{2}\sum_{i}^{N_T}\sum_{j}^{N_R}\int_0^{T_f}\left(E_c^s(t)\Big|_{ij} - E_m^s(t)\Big|_{ij}\right)^2 dt \tag{1}$$

where N_T is the number of transmitters, N_R is the number of receivers and T_f is the fixed final time. E_m^s is the measured scattered field and E_c^s is the calculated one at measuring point.

Applying the first variation to (1) with respect to inversion variable vector *{p}*, one can obtain the derivatives of error function as

$$\frac{dF}{d\{p\}} = \frac{1}{2}\sum_{i}^{N_T}\sum_{j}^{N_R}\int_0^{T_f}\frac{\partial G_{ij}}{\partial\{p\}} + \frac{\partial G_{ij}}{\partial E_c^s\big|_{ij}}\frac{\partial E_c^s\big|_{ij}}{\partial\{p\}}dt \tag{2}$$

where $G_{ij} = \left(E_c^s\big|_{ij} - E_m^s\big|_{ij}\right)^2$

In general, the scattered field variable E_c^s has an implicit relation with the variables *{p}* and $dF/d\{p\}$ can be obtained using an indirect method. To reduce the computing time, we introduce the adjoint variable method [4].

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B. FETD and Design Sensitivity Analysis

From the Maxwell's equations, one can derive the 2-D TM^z scalar wave equation as following

$$\nabla^2 e_z - \frac{\varepsilon_r}{c_0^2}\frac{\partial^2 e_z}{\partial t^2} - \mu_0 \sigma \frac{\partial e_z}{\partial t} = \mu_0 \frac{\partial J_z}{\partial t} \qquad (3)$$

where ε_r denotes relative permittivity, σ means conductivity and c_0 means velocity of light in free space. Applying the nodal element and Galerkin's formula to (3), one can discretize (3) and construct the matrix equation as

$$[K]\{e_z\} + [M]\{\ddot{e}_z\} + [B]\{\dot{e}_z\} = \{Q\} \qquad (4)$$

subject to

$$e_z(0) = 0, \quad \dot{e}_z(0) = 0 \qquad (5)$$

where dot denotes the time derivative. Matrices *[K]*, *[M]*, *[B]* and *{Q}* can be represented as

$$K_{ij}^e = \int_{\Omega^e} \nabla N_i \nabla N_j d\Omega^e \qquad (6a)$$

$$M_{ij}^e = \frac{1}{c_0^2}\int_{\Omega^e} \varepsilon_r^e N_i N_j d\Omega^e \qquad (6b)$$

$$B_{ij}^e = \mu_0 \int_{\Omega^e} \sigma^e N_i N_j d\Omega^e \qquad (6c)$$

$$Q_i^e = -\mu_0 \int_{\Omega^e} N_i \dot{J} d\Omega^e \qquad (6d)$$

Using the adjoint variable λ, one can derive the adjoint equation of (3) as following

$$[M]\{\ddot{\lambda}\} - [B]\{\dot{\lambda}\} + [K]\{\lambda\} = \left\{\frac{\partial G}{\partial E_c^s}\right\}^T \qquad (7)$$

subjected to

$$\lambda(T_f) = \dot{\lambda}(T_f) = 0 \qquad (8)$$

Equation (8) is terminal conditions on λ, for solving (7). To deal with the terminal conditions, the backward time scheme, $\tau = T_f - t$ is introduced. Then (7) can be converted into the initial-value problem. Using (2) and (7), one can transform the design sensitivity (2) into

$$\frac{\partial F}{\partial \varepsilon_r} = \sum_i^{N_T}\sum_j^{N_R}\int_0^{T_f} \lambda^T \frac{\partial}{\partial \varepsilon_r} R(t, \varepsilon_r, \sigma) dt \qquad (9a)$$

$$\frac{\partial F}{\partial \sigma} = \sum_i^{N_T}\sum_j^{N_R}\int_0^{T_f} \lambda^T \frac{\partial}{\partial \sigma} R(t, \varepsilon_r, \sigma) dt \qquad (9b)$$

where

$$R(t, \varepsilon_r, \sigma) = \{Q\} - [M]\{\tilde{\ddot{e}}_z\} - [B]\{\tilde{\dot{e}}_z\} - [K]\{\tilde{e}_z\} \qquad (10)$$

The notation ~ indicates that argument is held constant for the derivative process with respect to ε_r and σ. Note that *[M]* is the only matrix dependent on ε_r, *[B]* is only the matrix dependent on σ.

C. FDTD and Design Sensitivity Analysis

From the uniqueness theorem of solution, one can transform (7) in to the Maxwellian coupled curl equations as following .

$$\frac{\partial \lambda^{E_z}}{\partial y} = -\frac{\partial \lambda^{B_x}}{\partial t} \qquad (11a)$$

$$\frac{\partial \lambda^{E_z}}{\partial x} = \frac{\partial \lambda^{B_y}}{\partial t} \qquad (11b)$$

$$\frac{\partial \lambda^{Hy}}{\partial x} - \frac{\partial \lambda^{Hx}}{\partial y} - \sigma \lambda^{E_z} = \frac{\partial \lambda^{D_z}}{\partial t} + J_z^\lambda \qquad (11c)$$

subject to

$$\lambda^{E_z}(T_f) = \lambda^{H_x}(T_f) = \lambda^{H_y}(T_f) = 0 \qquad (12)$$

And these adjoint variable vectors satisfy the constitutive relation as the electromagnetic field vectors. That is,

$$\vec{\lambda}^D = \varepsilon \vec{\lambda}^E, \ \vec{\lambda}^B = \mu \vec{\lambda}^H \qquad (13)$$

In (7), J_z^λ is a pseudo electric current density and can be obtained from the relation of

$$\left.\frac{\partial G}{\partial E_z}\right|_{\Omega_m} = \mu_0 \int_{\Omega_m} N_i \dot{J}_z^\lambda d\Omega \qquad (14)$$

For a specific point in design domain, J_z^λ can be represented as a point current source following

$$J_z^\lambda(x, y, t) = J_z^\lambda(t)\delta(x - x_m, y - y_m) \qquad (15)$$

By inserting (15) into (14) and assuming that the grid is a square quadrilateral, the right-hand side of (14) is

$$\mu_0 \int_{\Omega_m} N_i \dot{J}_z^\lambda d\Omega = \mu_0 \dot{J}_z^\lambda \Delta \qquad (16)$$

where Δ is the area of grid. Therefore, J_z^{λ} can be written as

$$J_z^{\lambda}(t) = \frac{1}{\mu_0 \Delta} \int_0^{t} \frac{\partial G(t')}{\partial E_z} dt' \tag{17}$$

D. Topology Optimization

In order to smoothly reconstruct the complex permittivity profiles, topology optimization based on the normalized material density is introduced [6]. In the topology optimization, the test domain to be reconstructed is divided into small grids and the material composition of each grid is taken as design variable. By controlling the material composition of each grid, the unknown object relative permittivity and sigma can be reconstructed. The key concept of this method is how to treat the material composition in order to estimate the objective function, and reconstruct the final object shape. There are two methods to treat the material composition, the homogenization method and density method. Homogenization method provided solid material physical and mathematical basis for the calculation of the material properties of the composite, or intermediate materials. On the other hand, the density method takes the material density of each grid as the design variable and it does not concern the microstructure but the results only. In this paper, the density method is preferred.

To apply the density method to the reconstruction scheme, the normalized density vector of material *{p}* is introduced, each of element p_i takes the value between 0 and 1. Using the normalized density vector *{p}*, one can represent the complex permittivity as

$$\varepsilon_r(p_1) = (\varepsilon_{ro} - 1)p_1^h + 1 \quad ,0 < p_1 \le 1 \tag{18a}$$

$$\sigma(p_2) = \sigma_o p_2^h \quad ,0 < p_2 \le 1 \tag{18b}$$

where h is the exponent which defines the relationship between material property and normalized material density. The normalized material density p is defined at each grid in test domain. When p_i is 0, it means the permittivity is that of air, and when p_i is 1. it means that of solid material. The value of p_i between 0 and 1 corresponds to the intermediate material property. Inserting (18) into (9), one can rewrite the derivative error function F with respect to the normalized material density p as

$$\frac{\partial F}{\partial p_1} = h \sum_i^{N_T} \sum_j^{N_R} \int_0^{t_f} \lambda^T \frac{\partial}{\partial p_1} \left(-[M]\{\tilde{e}_z\}\right)(\varepsilon_{ro} - 1) p_1^{h-1} dt \tag{19a}$$

$$\frac{\partial F}{\partial p_2} = h \sum_i^{N_T} \sum_j^{N_R} \int_0^{t_f} \lambda^T \frac{\partial}{\partial p_2} \left(-[B]\{\tilde{e}_z\}\right)\sigma_o p_2^{h-1} dt \tag{19b}$$

Then, (11a)-(11c) can be also solved by using FDTD technique with terminal conditions (12). And introducing the electric fields and adjoint variables solved by using FDTD, one can calculate the design sensitivity also. As an optimization algorithm, the steepest descent method is used because of its simplicity.

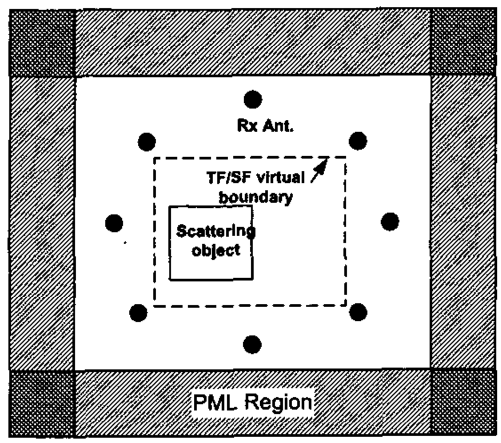

Fig. 1. Design configuration. The TM^z plane wave is incident at direction and at each Rx Ant. position E_z field is measured.

III. Numerical Example

In order to validate the proposed method, our method is applied to the 2-D reconstruction of dielectric object illuminated by the TM^z wave with a gaussian pulse. The analysis model is shown in Fig. 1. In order to realize a plane wave source, the total-field/scattered-field scheme was adopted. The incident wave is a gaussian pulse modulated by a sine function with center frequency of 5GHz. The number of transmitters is 16 and the number of receivers is 16. The measurement points are located in the scattered-field region around the central point of test domain. The number of grids in test domain is 60 by 60. Fig. 2(a) and 2(b) show the original ε_r and σ of presented model. The medium is composed of two concentric square cylinders. The inner cylinder with ε_r =2.5, σ =0.2 is surrounded by a cylinder with ε_r =2.0, σ =0.1 and the other region is filled with air. Fig. 2(c) and 2(d) show the reconstructed profiles which are obtained after 100 iterations.

IV. CONCLUSION

In this paper, we developed a numerical two-dimensional reconstruction algorithm for microwave imaging in TM^z case. The algorithm utilizes the FDTD, design sensitivity analysis and topology optimization technique. The method has been applied to the scattering objects that are illuminated by the pulse type wave source. The objects are successfully reconstructed in both of dielectric constant and electric conductivity.

ACKNOWLEDGEMENT

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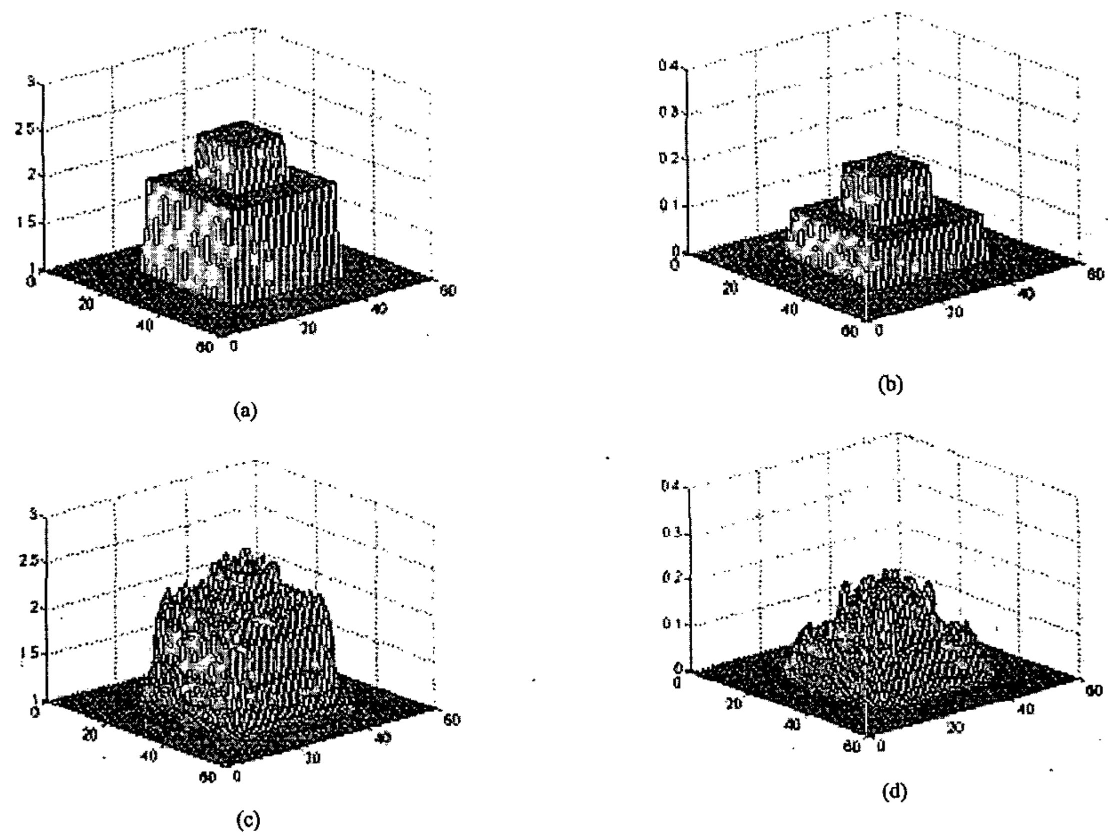

Fig. 2. Reconstruction result (a) original ε_r shape (b) original σ shape
(c) reconstructed ε_r (d) reconstructed σ